Comments on "Standing Waves on a Hanging Rope"

In his note on "Standing Waves on a Hanging Rope," Robert E. Gibbs[1] concludes that for this problem the standing-wave frequencies are odd harmonics. This is hard to understand since, in the formal theory, the solution for the amplitude of an oscillating rope that is vertically suspended is described by means of the zero-order Bessel function.

The author remarks that the distance of the first node from the bottom is noticeably greater than predicted by his calculation. When using the Bessel function method, however, the position of this node is correctly predicted. For the second standing-wave mode, for instance, this theory, in agreement with the experiment, localizes the node at $x = 0.190L$, where L is the total length of the rope. The localization of this node at $x = 0.111L$, as predicted by Gibbs, is clearly in contradiction with the formal theory and with the experimental observation.

I have the impression that the simple method suggested by the author in estimating the wave frequencies of the hanging rope is fundamentally wrong. In predicting these frequencies, he uses the relation $f = 1/T$ where T, the period of the vibration of the particles of the rope, is related to the time for the wave pulse to go from the bottom to the top of the rope. In my opinion this is not self-evident since the speed of the wave pulse is not constant.

The statement in the original note that the time between each pair of nodes interpreted as a half period is twice the time from the bottom to the first node is not correct. This can easily be shown by using the remark of the author himself that in the second mode the first node is closer to one-fifth of the way up from the bottom ($x = 0.200L$, in fact in agreement with the formal theory) than one-ninth ($x = 0.111L$ as predicted by him). Indeed, the time for the pulse to go from the bottom to position x is given by:

$$t_x = 2\sqrt{\frac{x}{g}} \qquad (1)$$

As for the first node where $x = 0.200L$, it follows that

$$t_x = 0.894\sqrt{\frac{L}{g}} \qquad (2)$$

For the second node $x = L$, which leads to

$$t_L = 2\sqrt{\frac{L}{g}} \qquad (3)$$

So, the time for the pulse to go from the first node to the second one is

$$t = t_L - t_x = 1.106\sqrt{\frac{L}{g}} \qquad (4)$$

which certainly does not correspond to $2t_x$.

My conclusion is that the approximation of Gibbs is not allowed for low resonant frequencies. His method possibly can be used for high standing-wave frequencies since it is a feature of the Bessel function that for large values of the argument the solutions tend to become sinusoidal. As a consequence, care must also be taken for the generalization of his method to a linear mass density of the form $\mu = kx^n$.

There exists, however, a very interesting problem with $n = -1/2$. This is the case for a vertically suspended Slinky. The speed of a wave pulse in a Slinky, calculated from Eq. (11) in Gibbs is given by

$$v = \sqrt{2gx} \qquad (5)$$

This is a remarkable result: it is the same as the free-fall formula observed by H. Mulder.[2]

Finally, I want to remark that the standing-wave frequencies of the hanging Slinky indeed form a series of odd harmonics. The behavior of these waves, however, is described by the standard wave equation instead of the Bessel differential equation. For further details I refer readers to the article of R.A.Young.[3]

References

1. R. E. Gibbs, *Phys. Teach.* **36**, 108 (1998).
2. H. Mulder, *NVON* **14**, 192 (1989), in Dutch.
3. R. A. Young, *Am. J. Phys.* **61**, 353 (1993).

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Response to Herreman

I thought it was clear from my article that I did not believe that my model was right, but only that it was instructive. Its value lies in the clarity with which it highlights the essential physics concepts with a minimum of mathematical difficulty. I took the Bessel function solution as presented by Satterly[1] to be correct and used it as a standard to specify the deficiencies in my own approach. The Bessel solution does not lead to odd harmonics, so that prediction of my model is incorrect. To be more explicit, all pre-