HYDROLOGICAL PROCESSES
*Hydrol. Process.* **16**, 2131–2140 (2002)
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/hyp.1146

# A numerical model of the river freezing process and its application to the Lena River

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## Abstract:

River freezing is a common phenomenon at high latitudes. The Lena River, one of the four largest rivers flowing into the Arctic Ocean, freezes over completely from early December to late April. The process of river-ice formation affects not only the river flow and local energy regime, but also influences hydrological and thermal conditions in the Arctic Ocean. In this study, a combined hydrological model used for a cold-region basin (Ma *et al*., 2000. *Hydrological processes* **14**: 639–651) was further developed to model hydrological processes in the Lena River. A simple method of accumulated degree-days was used in describing river-ice growth and decay. River flow routing was used to compute the hydrograph according to the depth of runoff stored in the river and the season. The combined model system was applied to the Lena River basin, using data for the period October 1986 to September 1987, and produced modelled hydrographs at six river sections that were an improvement on those resulting from the earlier study. Although the simulation duration in this study was limited to 1 year, it is expected that the model system will be an effective tool in making hydrological simulations in cold regions in the future. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS Lena River basin; river ice; breakup date; hydrological modelling

## INTRODUCTION

The Arctic Ocean and its marginal seas are key areas for understanding the global climate system and its dynamics. The present state of the Arctic Ocean itself and its effect on the global climate system depend strongly on the discharge into it of large rivers, which amounts to 10% of global runoff (ARCSS Workshop Steering Committee, 1990). Aagaard and Carmack (1989) indicated that approximately 80% of the total terrestrial runoff to the Arctic Ocean is supplied by the Ob, Yenisei, Lena and Mackenzie rivers. In these basins, the rivers and lakes are frozen for many months during the winter season. The Lena River is one of the three largest rivers in Siberia; it is 4400 km in length, flows from the southern mountainous regions to the northern lower-plain areas, and discharges into the Arctic Ocean (Figure 1). According to the GAME–Siberia (GEWEX Asian Monsoon Experiment in Siberia) project data set measured in 1986, 1987 and 1988, freezing starts near the outlet in late October, proceeds to the middle reaches in early November, and to the upstream parts in late November. Thawing starts at the upstream reaches in late April, and then extends into the middle and downstream regions. Downstream, river thawing occurs later than June. Hydrological simulation of the basin is one of the purposes of the GAME–Siberia project. However, the continental scale and severe climatic condition of the basin make the water cycle study very difficult. Ma *et al*. (2000) developed a hydrological simulation, using a combined model, in order to test their understanding of the hydrological processes in the Lena River basin. Using the model, the land surface processes (snowmelt, evapotranspiration, active layer dynamics of the permafrost, river flow formation, and so on) can be simulated. However, the calculated

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*Received 2 February 2001*
*Accepted 25 September 2001*

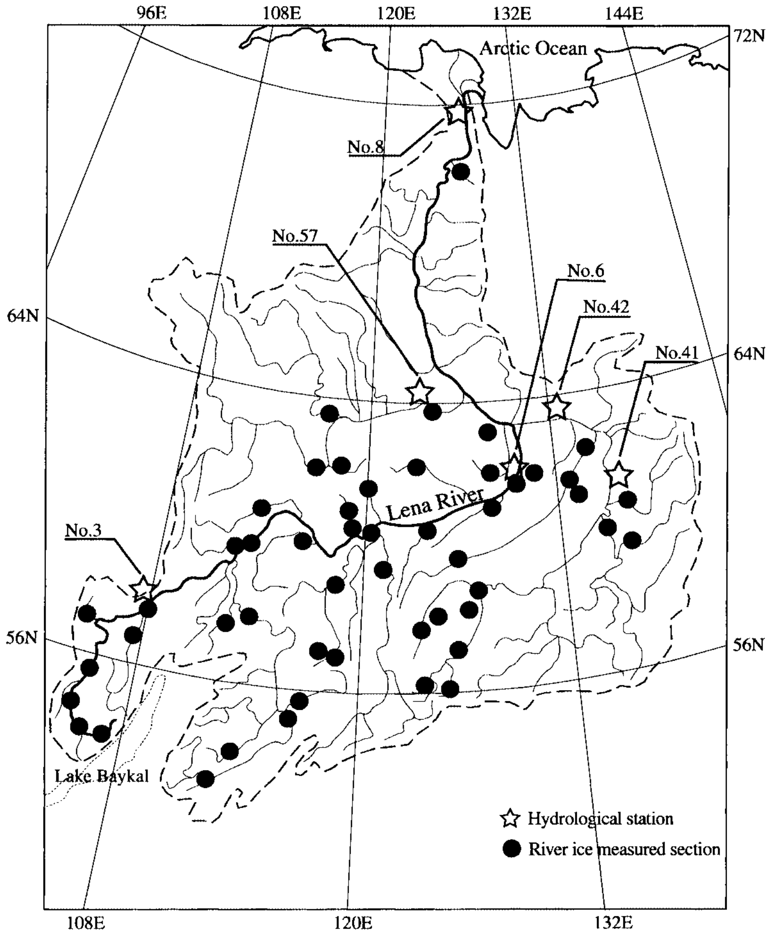


Figure 1. Map of the Lena River basin, showing the locations of hydrological stations and river sections where water temperature and ice depth measured

hydrographs peaked earlier than those observed, as river freezing processes were not considered (Ma *et al.*, 2000). River freezing occurs in high-latitude areas (e.g. Mackay, 1965; Johnson and Kistner, 1967; Nuttall, 1970; Slaughter and Samide, 1971; Sampson, 1973; McFadden and Collins, 1977; Parkinson, 1982; Shen *et al.*, 1991; Soldatova, 1992, 1993; Prowse, 1994; Scrimgeour *et al.*, 1994). Despite its significance to the environment and the economy, the study of river ice has advanced relatively slowly because of the complexity of river-ice phenomena (Beltaos, 2000). There are two basic methods of predicting ice phenomena in rivers (Michel *et al.*, 1986). The first uses past records (e.g. Laszloffy, 1948; Liser 1959; Deslauriers 1966; Gerard and Karpuk, 1979), and the second uses mathematical or empirical relations based on physical principles (e.g. Shulyakovskii, 1963; Williams, 1965; Michel, 1971; Ashton, 1973, 1979; Greene, 1981; Beltaos, 1984a). A more rigorous solution of the river-ice process must take into account the effects of snow cover, radiation, evaporation and convection in the air, and advection in the water; however, the climatic data required are seldom available. For most slow-flowing rivers, though, changes in the water and ice system progress slowly and, compared with other processes, can be regarded as being in a steady state. Thermal effects dominate ice growth and decay processes, and river-ice thickness can be predicted using local hydrological and meteorological data (e.g. air temperature, radiation and water temperature). Ma *et al.* (2001) conducted a river ice-cover process study, in which a simple, accumulated degree-day method was used to estimate

 

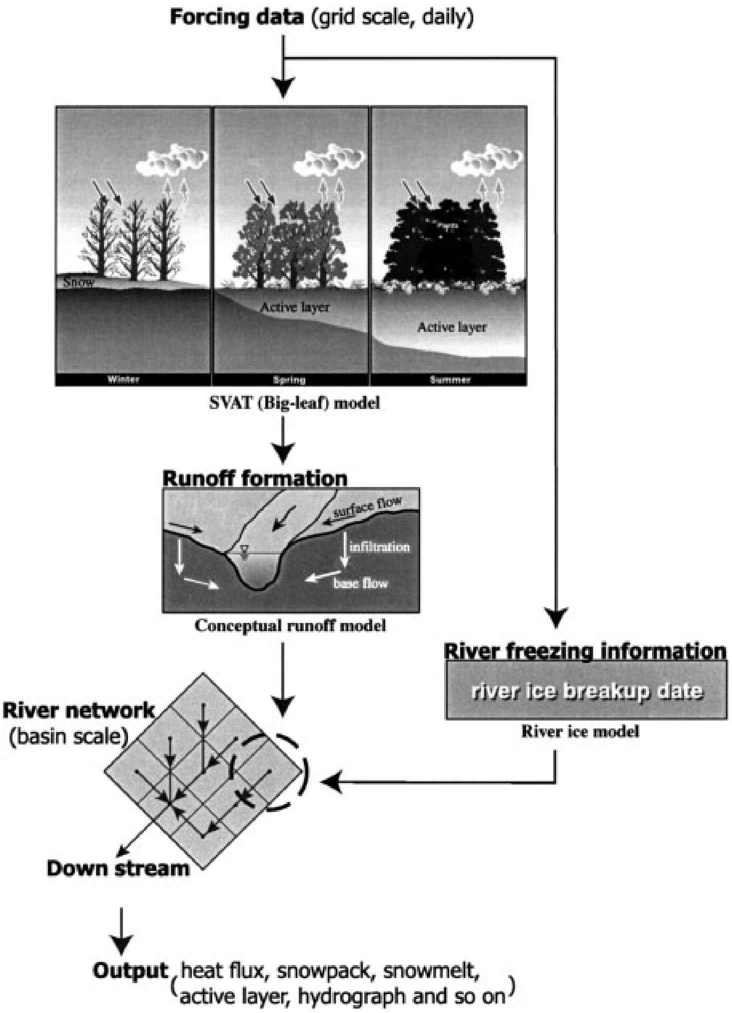


Figure 2. Flowchart of the modelling system

ice-cover growth and decay events using daily routine meteorological data. Their results showed that the breakup dates of 43 river sections over the Lena basin could be modelled adequately. However, a combined river-ice and hydrological simulation has never been attempted. In this study, a one-dimensional framework that considers the impact of river freezing processes on river flood formation is presented, which attempts to simulate hydrological processes in cold regions. The model is applied to the Lena River basin and the results are compared with those of Ma *et al.* (2000).

## DESCRIPTION OF THE MODEL STRUCTURE

The model is composed of four components: a one-dimensional soil–vegetation–atmosphere transfer (SVAT) model, a runoff-formation model, a river-ice model and a river-routing model. The SVAT model, runoff-formation model and river-routing model (without a freezing factor) were described in Ma *et al.* (2000), and the river-ice model was detailed in Ma *et al.* (2001). Figure 2 shows the model structure. The input of the model system is daily meteorological data and the outputs include heat fluxes on the land surface, depth of the permafrost active layer, the hydrograph at the watershed outlet, and so on. A brief summary of each model component is presented below.

*SVAT model*

The SVAT model is a simple biosphere model, in which the land surface is regarded as a 'big-leaf' to represent the vegetation and a soil layer. Using daily meteorological data, the model provides estimates of latent and sensible heat fluxes between the land surface and the atmosphere, and of thermal regimes in the snow-cover and soil layer. The heat fluxes on the land surface are determined by the Penman–Monteith equation, involving two parameters to describe the surface and its turbulence characteristics: the aerodynamic resistance $r_a$ and the surface resistance $r_s$. The value of $r_a$ is derived from the wind speed at an arbitrary reference height above the canopy by using the average wind profile parameters and the characteristics of the vegetation. For the snow-free season, the value of $r_s$ can be estimated from the equation of Blyth and Harding (1995). An important parameter in Blyth's equation, $r_{s\ min}$, is estimated from the linear relationship between $r_{s\ min}$ and the radiative dryness index, the ratio of annual net radiation to annual precipitation as an index of local soil conditions (Ma *et al.*, 1999). A pure implicit finite difference method is used to solve thermal conduction in the snowpack and in the upper layer of permafrost. The model input is routine daily meteorological data, including air temperature (minimum, maximum and average), precipitation, wind speed, relative humidity, atmospheric pressure, and sunshine duration. By using the SVAT model, heat fluxes, snowmelt, and the depth of the active layer of permafrost are derived. One of the outputs of the SVAT model, the effective rainfall (which is the total amount of rainfall and snowmelt, minus evapotranspiration), is divided into four components in the runoff model.

*Runoff model for each grid*

A conceptual model (Fukushima, 1988) is used to determine the formation of runoff for each $1^\circ \times 1^\circ$ grid box. It includes a reservoir system that represents each of the four runoff components: saturated land surface runoff, infiltration runoff from the topsoil zone, base runoff, and direct runoff from the water surface. In modelling the permafrost condition, a parameter-related mean effective soil depth is used as a function of the active layer depth (Ma *et al.*, 2000). The estimated runoff for each grid is then used as input data for the river-routing model.

*River-ice model*

A simple method using accumulated degree-days is used to determine river-ice growth and decay processes. This method is often used in river-ice studies (e.g. Greene, 1981; Michel *et al.*, 1986; Shen *et al.*, 1991). Here, we have assumed that the river ice expands upwards when the air temperature is continuously below the freezing point, and that the river-ice decay process begins when the growth process ends.

*Ice growth*

A commonly used set of boundary conditions takes air temperature as constant and equal to the surface temperature of the ice layer. Turbulent heat transfer between the water and ice is neglected, and it is assumed that there is no snow layer (no data are available on snow cover over river ice in our data set). Therefore, the heat flux between the ice and water can be rewritten as

$$\rho_i \lambda \frac{dZ_i}{dt} = \frac{K_i(T_m - T_s)}{Z_i} \tag{1}$$

where $\rho_i$ is the ice density, $\lambda$ is the latent heat of fusion, $Z_i$ is the ice thickness, $T_s$ and $T_m$ (0 °C) are the temperatures at the surface and bottom of the ice layer respectively, and $K_i$ is the thermal conductivity. Integration over time leads to the classic Stefan solution

$$Z_i = \left(\frac{2K_i}{\rho_i \lambda}\right)^{0.5} \left[\int_0^t (T_m - T_s)dt\right]^{0.5} \tag{2}$$

 

*Ice decay*

An integrated form of the heat-balance equation developed by Ashton (1973), which considers the heat flux between ice and water for a time interval, is shown as follows

$$\Delta t = \frac{-\rho_i \lambda (Z_t - Z_o)}{Q_w} - \frac{K_i \rho_i \lambda (T_m - T_s)}{(Q_w)^2} \ln \left[ \frac{1 - \dfrac{Q_w Z_t}{K_i (T_m - T_s)}}{1 - \dfrac{Q_w Z_o}{K_i (T_m - T_s)}} \right] \tag{3}$$

where $\Delta t$ is the time interval, and $Z_t$ and $Z_o$ are respectively the final and initial values of the ice thickness during the period of the time step. The value for the ice depth at the end of the time step is calculated from the equation. $Q_w$ is the flux of heat from the water to the ice, and was defined by Ashton (1979) as:

$$Q_w = B_i \frac{U_w^{0\cdot 8}}{Z_w^{0\cdot 2}} (T_w - T_m) \tag{4}$$

where $B_i$ is an empirical constant, and $U_w$, $Z_w$, and $T_w$ are the flow velocity, the water depth and the water temperature respectively. However, water temperature data are not commonly available for the ice-cover season for a large basin, and it is therefore assumed that the water temperature increases when the local daily air temperature is over 0 °C. Therefore, it is possible to use numerical methods to find an approximate solution for the ice thickness $Z_t$ that represents the value after the given period $\Delta t$, because the time interval is specified and other variables are either input data or can be estimated.

The model was applied to the Lena River basin (Ma *et al.*, 2001) and tested against a 10-day data set for ice thickness during the freeze-up period, including 51 sections of the rivers in the basin (Figure 1). Table I shows the ice thickness and water temperature data observed at the Tabaga (No. 6 in Figure 1) from 1986 to 1987 as an example for the data set. It was assumed that: (1) the temperature $T_m$ at the bottom of the ice layer was zero; (2) the water temperature $T_w$ increased by 0·05 °C day$^{-1}$ (from zero) during the river-ice decay period, in accordance with observations (Greene, 1981; Parkinson, 1982). For the basin, the value of $U_w$ was set to 0·4 m s$^{-1}$ (Ma *et al.*, 2000), and $Z_w$ to 10 m (Ma *et al.*, 2001). The results showed that the estimated breakup dates were consistent with those observed in about 60% of all river sections. Figure 3 shows the interrelationship of the observed and estimated breakup dates for all sections. Most differences are of less than 10 days, indicating the success of the simulation. Therefore, this model has been accepted in this study. Figure 4 shows the estimated spatial distribution of breakup dates over the basin in 1987.

*River routing model*

A constant velocity of 0·4 m s$^{-1}$ was assumed for the river network of the Lena River basin with a 0·1° × 0·1° grid (Ma *et al.*, 2000). Actually, the water flows in the channel system are very complex, especially

Table I. Ice thickness and water temperature data observed at the Tabaga from 1986 to 1987

| Day | Ice thickness (cm) | | | | | | | | Water temperature (°C) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Apr. | May | Apr. | May | Jun. | Jul. | Aug. | Sep. | Oct. |
| **1986** | | | | | | | | | | | | | | | |
| 10 | — | 23 | 51 | 78 | 93 | 115 | 127 | 121 | | 9·4 | 19·0 | 18·6 | 12·5 | 3·7 | |
| 20 | — | 45 | 56 | 82 | 96 | 123 | 126 | — | | 13·8 | 21·7 | 17·0 | 9·8 | 0·6 | |
| Last day | — | 48 | 70 | 90 | 98 | 126 | 123 | — | — | 16·3 | 17·3 | 14·6 | 6·6 | 0·0 | |
| **1987** | | | | | | | | | | | | | | | |
| 10 | — | 19 | 53 | 90 | 108 | 119 | 124 | 119 | | — | 11·6 | 17·4 | 17·5 | 13·2 | 5·4 |
| 20 | — | 45 | 70 | 96 | 113 | 120 | 124 | — | | 0·2 | 14·5 | 16·1 | 15·9 | 11·2 | 0·9 |
| Last day | — | 50 | 78 | 103 | 117 | 123 | 124 | — | — | 4·5 | 16·6 | 19·7 | 14·9 | 8·7 | 0·0 |

 

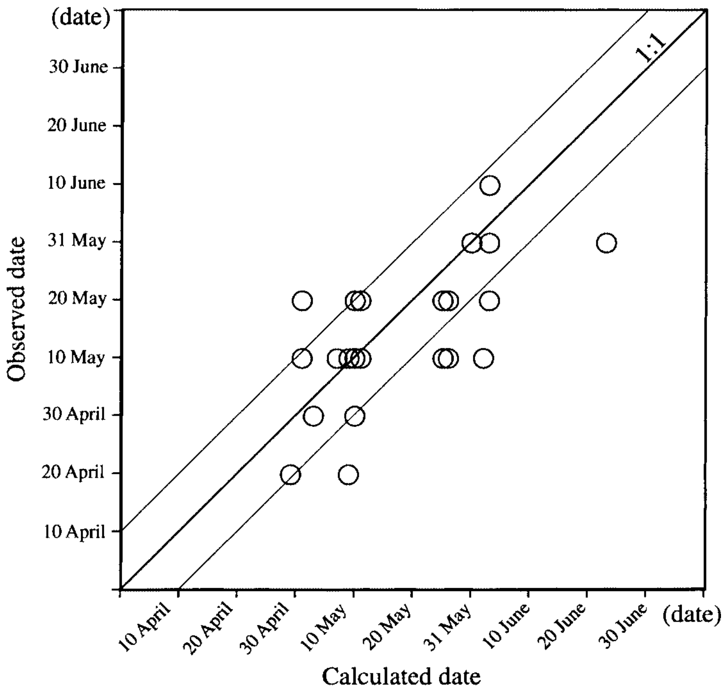


Figure 3. Estimation of river-ice-breakup date versus that of observed at the 43 river sections in the Lena River basin in 1987 (eight of the 51 sections are missing for that year)

during the river-ice breakup season. Here, it is assumed that most of the snowmelt is stored temporarily in the river while the river fully freezes. Once the river ice breaks up, part of the stored runoff over a given depth is moved along the river with high velocity (e.g. Doyle and Andres, 1978; Parkinson, 1982; Beltaos, 1984b; Gerard *et al.*, 1984; Prowse, 1994), taking account of the hydraulic pressure from snowmelt stored in the channel. Here, the relationship of storage and runoff in the channel is described for a non-linear reservoir (Fukushima, 1988) as follows:

$$S = KR^{0.6} \tag{5}$$

where $S$ (mm) is the depth of storage of snowmelt for a grid unit, $R$ (mm day$^{-1}$) is runoff and $K$ (mm$^{0.4}$ day$^{0.6}$) is a constant. Velocity is assumed as varying with the storage depth and season. It can be described as follows: in the river-freezing season

$$\begin{cases} S = K_1 R^{0.6} \\ u = v_1 \end{cases} \tag{6}$$

in other seasons

$$\begin{cases} S = K_2 R^{0.6} \\ u = v_2 \end{cases} \qquad S \geq h \text{ mm} \tag{7}$$

and

$$\begin{cases} S = R \\ u = v_3 \end{cases} \qquad S < h \text{ mm} \tag{8}$$

where $u$ is flow velocity, $h$ is depth of storage.

Using these procedures, a simulated hydrograph at a watershed outlet can be derived.

## APPLICATION AND RESULTS

The model was applied to the Lena River basin. The duration of application was set to be the same period as that of Ma *et al*. (2000), which was from 1 October, 1986 to 30 September 1987. For this study, two data sets were required: (1) a runoff data set and (2) a river-ice breakup-date data set for each grid element over the basin. Here, the runoff data set for a $1^\circ \times 1^\circ$ grid calculated by Ma *et al*. (2000) was used to link the SVAT model and the runoff formation model. The spatial distribution of the river-ice breakup date over the basin with a $1^\circ \times 1^\circ$ grid size (Figure 4) was estimated using the river-ice model and $1^\circ \times 1^\circ$ grid meteorological data derived by Ma *et al*. (2000) over the Lena River basin. These two data sets were input into the river routing model.

The parameters used in Equations (6)–(8) were decided according to the characteristics of the hydrological processes in the basin and the test run results of the model. $K_1$ and $K_2$ were set as 150 mm$^{0\cdot4}$ day$^{0\cdot6}$ and 20 mm$^{0\cdot4}$ day$^{0\cdot6}$ respectively for the river-ice-cover period and river-breakup period, both of which exceed the value of 2 mm$^{0\cdot4}$ day$^{0\cdot6}$ used by Fukushima (1988) and Ma *et al*. (2000). This means that the water resistance to forward movement during the period when ice floes are present is larger than that during the ice-free season. The storage depth $h$ was set to 40 mm. Meanwhile, $v_1$ and $v_3$ were both set to 0·4 m s$^{-1}$ (Ma *et al*., 2000), and $v_2$ was set to 0·7 m s$^{-1}$ when the storage depth was greater than 40 mm, because after ice breakup the hydraulic pressure from upstream will cause part of the runoff stored in the river to flow rapidly.

Figure 5 shows the daily hydrographs calculated by Ma *et al*. (2000) at six hydrological gauges. Station Nos 3, 6 and 8 are the main river sections; they are located upstream, in the middle reaches, and downstream. The other sections (Nos 41, 42 and 57) are located in the three tributary rivers. Figure 6 shows the results of this study considering the river freezing process. Comparison of Figures 5 and 6 shows that many aspects

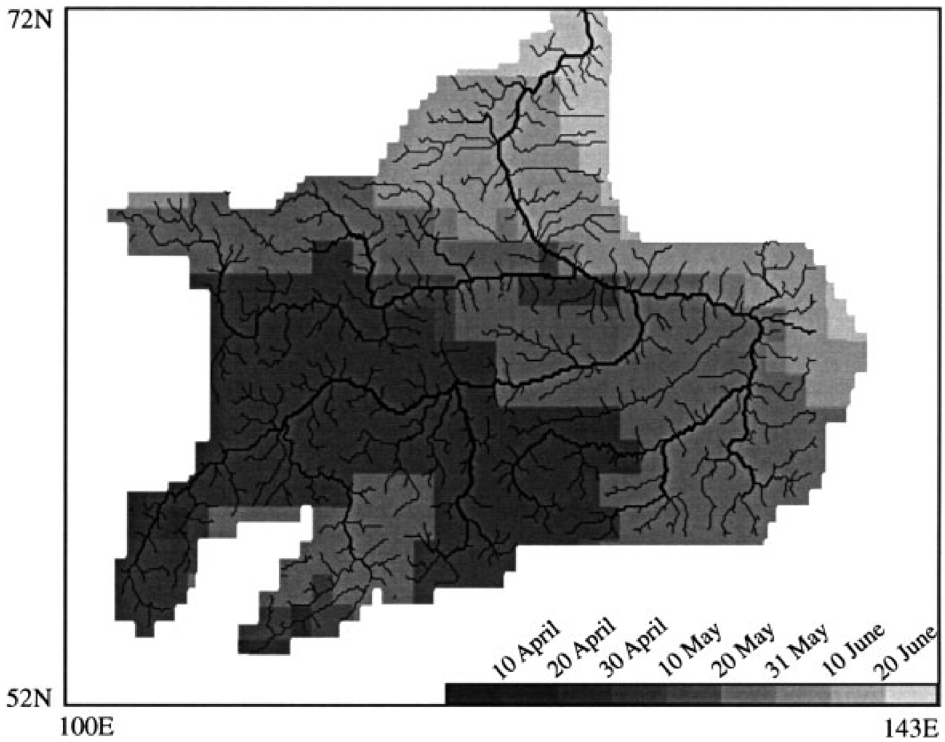


Figure 4. Estimated spatial distribution of river-ice-breakup date in the Lena River basin in 1987

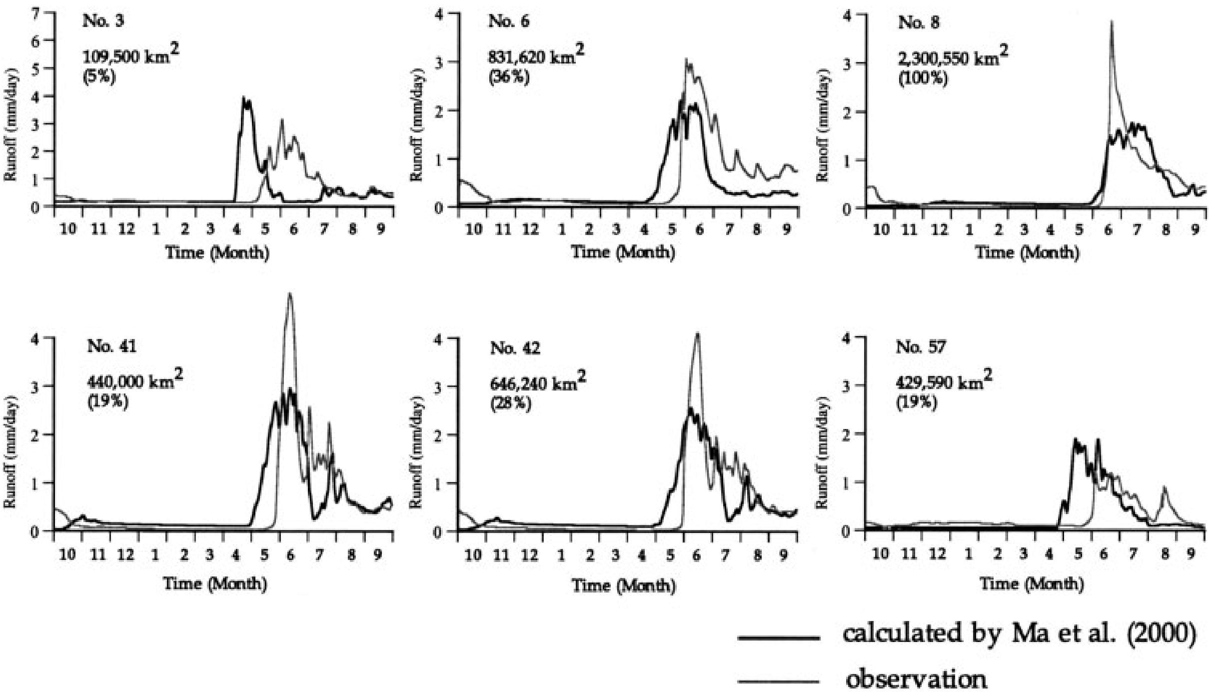


Figure 5. Observed and calculated values for daily runoff obtained by Ma *et al*. (2000) for six hydrological stations in the Lena River basin from October 1986 to September 1987. The number alongside each station indicates the watershed area and, in brackets, the percentage of the whole area of the Lena River basin

of the simulation have been improved: (1) the timing of flood rise is close to that observed for all stations; (2) the estimated flood peak corresponds with that observed, with the exception of No. 3; (3) the calculated hydrograph is more reasonable for all stations. An evaluation of root-mean-square error (RMSE) for each simulation is shown in Table II. When compared with the results of Ma *et al*. (2000), it is clear that the model used in this study is the more accurate.

The hydrograph for No. 3 in Figure 6 shows a calculated flood peak that is larger than that observed. The watershed area of No. 3 is the smallest of the six, and occupies 5% of the basin. The input data were derived from only 40 meteorological stations, and at a grid resolution of $1^\circ \times 1^\circ$ this may introduce interpolation errors, especially for small basins. However, the result for No. 57 in Figure 6 looks reasonable, allowing for the fact that the river is controlled by an upstream dam, the effect of which was not considered in this study. Moreover, river flow during the ice breakup period is also influenced by some human activities, such as the

Table II. Values of RMSE for the Lena River hydrological simulation and rates of error improvement compared with the results of Ma *et al.* (2000)

| Station | No. 3 | No. 6 | No. 8 | No. 41 | No. 42 | No. 57 |
|---|---|---|---|---|---|---|
| RMSE1[a] | 0·9815 | 0·5316 | 0·3314 | 0·6395 | 0·4743 | 0·4266 |
| RMSE2[b] | 0·9282 | 0·3995 | 0·2577 | 0·4781 | 0·3929 | 0·3842 |
| $\Delta R$/RMSE1 (%)[c] | 5·4 | 24·8 | 22·2 | 25·2 | 17·2 | 9·9 |

[a] RMSE1 = RMSE of the result of Ma *et al.* (2000).
[b] RMSE2 = RMSE of this study.
[c] $\Delta R$ = RMSE1 − RMSE2.

 

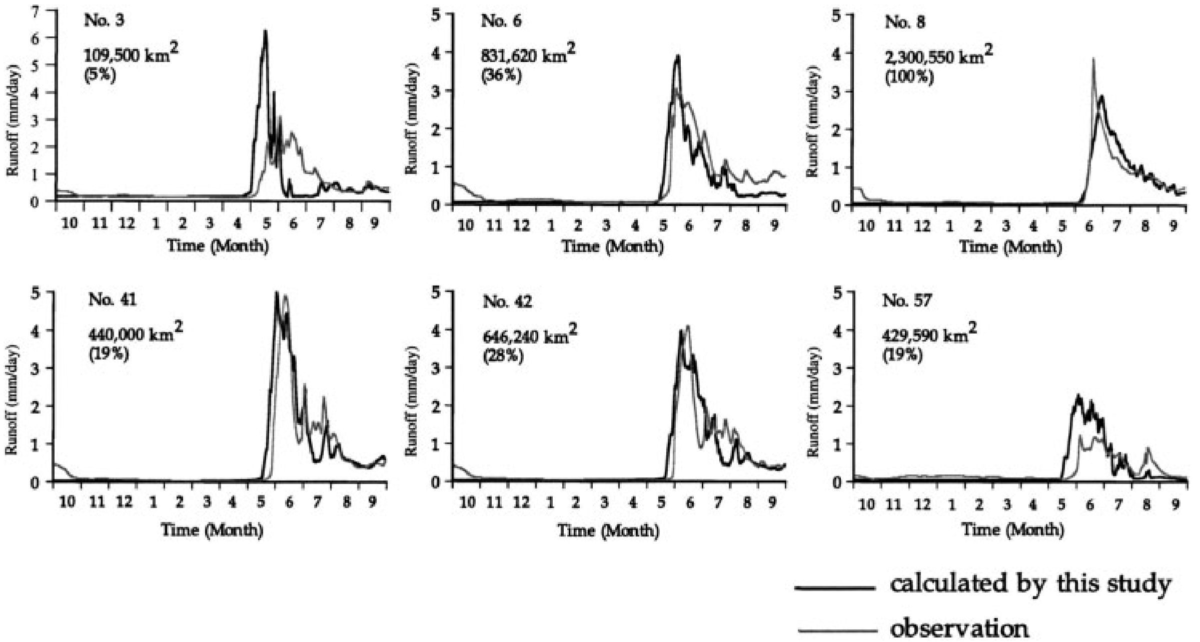


Figure 6. Observed and calculated values for daily runoff obtained in this study for six hydrological stations in the Lena River basin from October 1986 to September 1987. Annotations as for Figure 5

use of explosives to reduce flooding. The preliminary results show that the model system is effective for simulation of the hydrological processes of cold-region river basins.

## CONCLUSIONS

A hydrological model for cold regions was developed from an earlier model (Ma *et al.*, 2000) by adding the river freezing process to the simulation. A simple river-ice model was used to estimate ice growth and decay processes. The output provided by this model was the river-ice breakup date, which was in turn used by a river-routing model to calculate the river-flow formation process. In the river-routing model, the flow velocity depended on the depth of storage of runoff stored in the river. When the depth of runoff for a grid element was over 40 mm the velocity of the flow was set to 0·7 m $s^{-1}$, otherwise it was set to 0·4 m $s^{-1}$, as in the earlier study. The model system was applied to the Lena River basin using the estimated runoff (Ma *et al.*, 2000) and estimated river-ice-breakup dates over the basin. The results show improved modelled hydrographs (at six stations from 1 October 1986 to 30 September 1987) compared with those of Ma *et al.* (2000). Predictions of flood timing were more accurate at all stations. The estimated hydrographs were reasonable for most of the stations, not only in terms of the flood peak but also in terms of the sharpness of the hydrographs. Although the simulation was limited to 1 year duration in this study, the model system is expected to prove an effective tool in making long-term hydrological simulations of the Lena River in the future.

### ACKNOWLEDGEMENTS

We thank T. Yasunari and T. Ohata for discussions and comments. This study was carried out as part of the GAME–Siberia activities under a grant from the Ministry of Education, Science, Sports and Culture of Japan. All data sets used in this study were provided by the GAME–Siberia committee.

 

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